Collect the problems of Jack Garfunkel

Mathematics and friend forum

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1. AMM 6300 by Jack Garfunkel

Show that if two numbers are chosen at random from the Fibonacci sequence $1, 1, 2, 3, 5, \ldots$, the probability P that they will be relatively prime satisfies the inequalities $8/\pi^2 > P > 7/\pi^2$.

2. AMM E2181 by Jack Garfunkel

Given any triangle ABC and a given segment BP on side BC, determine (by geometric construction) segments CQ, AT on sides CA and AB respectively, so that equilateral triangles erected outwardly on these three segments have vertices that are the vertices of an equilateral triangle.

3. AMM E2634 by Jack Garfunkel

Let A_i, $i = 0, 1, 2 \pmod 3$ be the vertices of a triangle, and let Γ be its inscribed circle with center O. Let B_i be the intersection of the segment A_iO with Γ and let C_i be the intersection of the line A_iO with the side $A_{i-1}A_{i+1}$.

Prove that

$$\sum A_iC_i \leq 3 \sum A_iB_i.$$

4. AMM E2715 by Jack Garfunkel

Let G be the centroid of the triangle $A_1A_2A_3$ and let

$$\theta_i = \angle\left(\overrightarrow{A_iA_{i+1}}, \overrightarrow{A_iG}\right), \qquad i = 1, 2, 3.$$

Prove or disprove that $\sum \sin\theta_i \leq 3/2$.

5. AMM E2716 by Jack Garfunkel

Let ABC be a triangle with P an interior point. Let A', B', and C' be the points where the perpendiculars drawn from P meet the sides of ABC. Let A'', B'', and C'' be the points where the lines joining P to A, B, and C meet the corresponding sides of ABC. Prove or disprove that

$$A'B' + B'C' + C'A' \leq A''B'' + B''C'' + C''A''.$$

6. AMM E2906 by Jack Garfunkel

Let I be the incenter of triangle ABC. Points A', B', C' are the intersections of AI, BI, CI with the incircle of ABC. Continue the process by defining I' (incenter of $A'B'C'$), then $A''B''C''$, etc. Prove that the angles of triangle $A^{(n)}B^{(n)}C^{(n)}$ approach $\pi/3$.

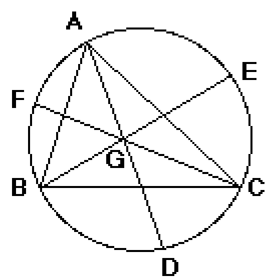
7. AMM E2924 by Jack Garfunkel

Triangle $A_1A_2A_3$ is inscribed in a circle; the medians through A_1 $[A_2]$ meet the circle again at M_1 $[M_2]$. The angle bisectors through A_1 $[A_2]$ meet the circle again at T_1 $[T_2]$. Prove or disprove: $|A_1M_1 - A_2M_2| \leq |A_1T_1 - A_2T_2|$.

8. AMM E2959 by Jack Garfunkel

Triangle ABC is inscribed in a circle. The medians of the triangle intersect at G and are extended to the circle to points D, E and F. Prove:

$$AG + BG + CG \leq GD + GE + GF.$$



9. AMM S23 by Jack Garfunkel and Leon Bankoff

Prove that the sum of the distances from the incenter of a triangle ABC to the vertices does not exceed half of the sum of the internal angle bisectors, each extended to its intersection with the circumcircle of triangle ABC.

10. CRUX 168 by Jack Garfunkel

If a, b, c are the sides of a triangle ABC, t_a, t_b, t_c are the angle bisectors, and T_a, T_b, T_c are the angle bisectors extended until they are chords of the circle circumscribing the triangle ABC, prove that

$$abc = \sqrt{T_aT_bT_ct_at_bt_c}.$$

11. CRUX 1049 by Jack Garfunkel

Let ABC and $A'B'C'$ be two nonequilateral triangles such that $A \geq B \geq C$ and $A' \geq B' \geq C'$. Prove that

$$A - C > A' - C' \iff \frac{s}{r} > \frac{s'}{r'},$$

where s, r and s', r' are the semiperimeter and inradius of triangles ABC and $A'B'C'$, respectively.

12. CRUX 1067 by Jack Garfunkel

(a)* If $x, y, z > 0$, prove that

$$\frac{xyz(x+y+z+\sqrt{x^2+y^2+z^2})}{(x^2+y^2+z^2)(yz+zx+xy)} \le \frac{3+\sqrt{3}}{9}.$$

(b) Let r be the inradius of a triangle and r_1, r_2 and r_3 the radii of its three Malfatti circles. Deduce from (a) that

$$r \le (r_1+r_2+r_3) \cdot \frac{(3+\sqrt{3})}{9}.$$

13. CRUX 1077 by Jack Garfunkel

For $i = 1, 2, 3$, let C_i be the center and r_i the radius of the Malfatti circle nearest A_i in triangle $A_1A_2A_3$. Prove that

$$A_1C_1 \cdot A_2C_2 \cdot A_3C_3 \ge \frac{(r_1+r_2+r_3)^3 - 3r_1r_2r_3}{3}.$$

When does equality occur?

14. CRUX 1083 by Jack Garfunkel

Prove the inequality

$$\sum \cos\frac{B-C}{2} \le \frac{2}{\sqrt{3}} \sum \cos\frac{A}{2},$$

where the sums are cyclic over the angles A, B and C of a triangle.

15. CRUX 1093 by Jack Garfunkel

Prove that

$$\left\{\frac{\sum \sin A}{\sum \cos\left(\frac{A}{2}\right)}\right\}^3 \ge 8 \prod \sin\frac{A}{2},$$

where the sums and product are cyclic over the angles A, B and C of a triangle. When does equality occur?

16. CRUX 1106 by Jack Garfunkel

The directly similar triangles ABC and DEC are both right-angled at C. Prove that

(a) $AD \perp BE$;

(b) AD/BE equals the ratio of similitude of the two triangles.

17. CRUX 1113 by Jack Garfunkel

Consider two concentric circles with radii r and $2r$, and a triangle ABC inscribed in the inner circle. Points A', B' and C' on the outer circle are determined by extending AB to B', BC to C', and CA to A'. Prove that the perimeter of triangle $A'B'C'$ is at least twice the perimeter of ABC. Equality is attained when ABC is equilateral.

18. CRUX 1125 by Jack Garfunkel

If A, B, and C are the angles of an acute triangle ABC, prove that

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\le\frac{3}{2}(\csc 2A+\csc 2B+\csc 2C)$$

with equality when triangle ABC is equilateral.

19. CRUX 1135 by Jack Garfunkel

(a) Given equilateral triangles ABC and $A'B'C'$ in the same plane, both labeled counterclockwise, prove that triangle $M_1M_2M_3$ is equilateral, where M_1, M_2 and M_3 are the midpoints of AA', BB', CC' respectively.

(b) Given similar triangles ABC and $A'B'C'$ in the same plane, prove that triangle $M_1M_2M_3$ is similar to triangle ABC, where M_1, M_2 and M_3 are as in (a).

20. CRUX 1150 by Jack Garfunkel

In a figure, $\triangle M_1M_2M_3$ and the three circles with centers O_1, O_2, O_3 represent the Malfatti configuration. Circle O is externally tangent to these three circles and the sides of triangle $G_1G_2G_3$ are each tangent to O and one of the smaller circles. Prove that

$$P(\triangle G_1G_2G_3)\ge P(\triangle M_1M_2M_3)+P(\triangle O_1O_2O_3),$$

where P stands for perimeter. Equality is attained when $\triangle O_1O_2O_3$ is equilateral.

21. CRUX 1151 by Jack Garfunkel

Prove or disprove that for an obtuse triangle ABC,

$$m_a+m_b+m_c\le s\sqrt{3}$$

where m_a, m_b, m_c denote the medians to sides a, b, c and s denotes the semiperimeter of $\triangle ABC$. Equality is attained in the equilateral triangle.

22. CRUX 1179 by Jack Garfunkel

Squares are erected outwardly on each side of a quadrilateral $ABCD$.

(a) Prove that the centers of these squares are the vertices of a quadrilateral $A'B'C'D'$ whose diagonals are equal and perpendicular to each other.

(b)* If squares are likewise erected on the sides of $A'B'C'D'$, with centers A'', B'', C'', D'', and this procedure is continued, will quadrilateral $A^{(n)}B^{(n)}C^{(n)}D^{(n)}$ tend to a square as n tends to infinity?

23. MM 936 by Jack Garfunkel

It is known that

$$h_a + h_b + h_c \le \sqrt{3}s,$$

where the h's represent altitudes to sides a, b, and c and s represents the semiperimeter of triangle ABC. Prove or disprove the stronger inequality

$$t_a + t_b + m_c \le \sqrt{3}s,$$

where the t's are the angle bisectors and m_c is the median to side c.

24. MM Q646 by Jack Garfunkel

If a, b, and c are the sides of a triangle ABC, and if t_a, t_b, and t_c are the angle bisectors of this triangle, and if T_a, T_b, and T_c are these angle bisectors extended until they become chords of the circle circumscribed about the triangle, then prove that $abc = \sqrt{T_aT_bT_ct_at_bt_c}$.

25. MSJ 532 by Jack Garfunkel

Let h_a, h_b, and h_c be the altitudes of $\triangle ABC$ and denote by H_a, H_b, and H_c the segments obtained by extending the altitudes to the circumcircle of $\triangle ABC$.

Calculate

$$\frac{H_a}{h_a} + \frac{H_b}{h_b} + \frac{H_c}{h_c}.$$

26. PME 331 by Jack Garfunkel

In a right triangle ABC, $A = 60°$ and $B = 30°$, with D, E, F the points of trisection nearest A, B, C on the sides AB, BC and CA respectively. Extend CD, AE and BF to intersect the circumcircle (O) at points P, Q, R. Show that triangle PQR is equilateral.

27. PME 341 by Jack Garfunkel

Prove that the following construction trisects an angle of $60°$. Triangle ABC is a $30° - 60° - 90°$ right triangle inscribed in a circle. Median CM is drawn to side AB and extended to M' on the circle. Using a marked straightedge, point N on AB is located such that CN extended to N' on the circle makes $NN' = MM'$. Then CN trisects the $60°$ angle ACM.

28. PME 351 by Jack Garfunkel

Angle A and angle B are acute angles of a triangle ABC. If $\angle A = 30°$ and h_a, the altitude issuing from A, is equal to m_b, the median issuing from B, find angles B and C.

29. PME 368 by Jack Garfunkel

Given is a triangle ABC with its inscribed circle (I). Lines AI, BI, CI cut the circle in points D, E, F respectively. Prove that

$$AD + BE + CF \ge \frac{\partial DEF}{\sqrt{3}}.$$

30. PME 374 by Jack Garfunkel

In a triangle ABC inscribed in a circle (O), angle bisectors AT_1, BT_2, CT_3 are drawn and extended to the circle with T_i lying on the circle. Perpendiculars T_1H_1, T_2H_2, T_3H_3 are drawn to sides AC, BA, CB respectively. Prove that

$$T_1H_1 + T_2H_2 + T_3H_3 \leq 3R,$$

where R is the radius of the circumcircle.

31. PME 387 by Jack Garfunkel

On the sides AB and AC of an equilateral triangle ABC, mark the points D and E respectively, such that $AD = AE$. Erect directly similar equilateral triangles CDP, AEQ, BAR on CD, AE, and AB respectively. Show that triangle PQR is equilateral. Also show that the midpoints of PE, AQ, and RD are vertices of an equilateral triangle.

32. PME 399 by Jack Garfunkel

Show that

$$\arcsin\left(\frac{x-3}{3}\right) + 2\arccos\sqrt{\frac{x}{6}} = \frac{\pi}{2}\ , \qquad (3 \leq x \leq 6).$$

33. PME 422 by Jack Garfunkel

Let perpendiculars be erected outwardly at A and B of a right triangle ABC $(C = 90°)$, and at M, the midpoint of AB. Extend these perpendiculars to points P, Q, R such that

$$AP = BQ = MR = \frac{AB}{2}\ .$$

Show that triangle PQR is perspective with triangle ABC.

34. PME 431 by Jack Garfunkel

In a right triangle ABC, with sides a, b, and hypotenuse c, show that $4(ac + b^2) \leq 5c^2$.

35. PME 442 by Jack Garfunkel

Show that the sum of the perpendiculars from the circumcenter of a triangle to its sides is not less than the sum of the perpendiculars drawn from the incenter to the sides of the triangle.

36. PME 453

Given two intersecting lines and a circle tangent to each of them, construct a square having two of its vertices on the circumference of the circle and the other two on the intersecting lines.

37. PME 473 by Jack Garfunkel

In an acute triangle ABC with angle $A = 60°$, P is a point within the triangle. Let D and E be the feet of the cevians through P from C and B respectively.

(a) If $BD = DE = EC$, prove that $AP = BP = CP$.
(b) If $AP = BP = CP$, prove that $BD = DE = EC$.
(c) If $\angle PBC = \angle PCB = 30°$, then $BD = DE = EC$.

38. PME 476 by Jack Garfunkel

If A, B, C and D are the internal angles of a convex quadrilateral, that is, if $A + B + C + D = 360°$, then

$$\sqrt{2}[\cos(A/2) + \cos(B/2) + \cos(C/2) + \cos(D/2)] \\ \leq [\cot(A/2) + \cot(B/2) + \cot(C/2) + \cot(D/2)],$$

with equality when $A = B = C = D = 90°$.

39. PME 492 by Jack Garfunkel

Given an acute triangle ABC with altitudes denoted by h_a, h_b and h_c and medians by m_a, m_b and m_c to sides a, b and c, respectively. The points P, Q and R are determined by the intersections $m_a \cap h_b$, $m_b \cap h_c$ and $m_c \cap h_a$, respectively. Prove

$$\frac{AP}{PL} + \frac{BQ}{QM} + \frac{CR}{RN} \geq 6,$$

where L, M and N are the feet of the medians.

40. PME 509 by Jack Garfunkel

Given a triangle ABC with its incircle I, touching the sides of the triangle at points L, M and N. Let P, Q and R be the midpoints of arcs NL, LM and MN, respectively. Form triangle DEF by drawing tangents to the circle at P, Q and R. Prove that the perimeter of triangle DEF does not exceed the perimeter of triangle ABC.

41. PME 512 by Jack Garfunkel

Denote the number of ways a positive integer n can be partitioned into 3 positive integers by $P_3(n)$. Prove that if a, b and c are positive integers and $a^2 + b^2 = c^2$, then

$$P_3(a) + P_3(b) = P_3(c).$$

42. PME 515 by Jack Garfunkel

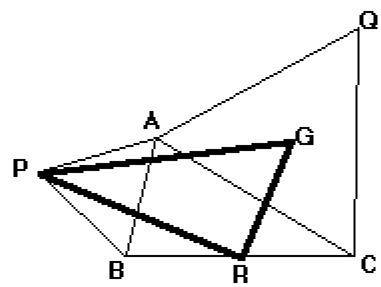
Given a sequence of concentric circles with a triangle ABC circumscribing the outermost circle. Tangent lines are drawn from each vertex of ABC to the next inner circle, forming the sides of triangle $A'B'C'$. Tangents are now drawn from vertices A', B' and C' to the next inner circle and they are the sides of triangle $A''B''C''$, and so on. Prove that the angles of triangle $A^{(n)}B^{(n)}C^{(n)}$ approach $\pi/3$.

43. PME 544 by Jack Garfunkel

Show that a quadrilateral $ABCD$ with $AD = BC = s$ and $\angle A + \angle B = 120°$ has maximum area if it is an isosceles trapezoid.

44. PME 553 by Jack Garfunkel

Given a triangle ABC, erect equilateral triangles BAP and ACQ outwardly on sides AB and CA. Let R be the midpoint of side BC and let G be the centroid of triangle ACQ. Prove that triangle PRG is a $30° - 60° - 90°$ triangle.



45. PME 572 by Jack Garfunkel

Let $ABCD$ be a parallelogram and construct directly similar triangles on sides AD and BC, and diagonals AC and BD. Triangles ADE, ACH, BDF and BCG are the directly similar triangles. What restrictions on the appended triangles are necessary for $EFGH$ to be a rhombus?

46. PME 584 by Jack Garfunkel

Let ABC be any triangle with base BC. Let D be any point on side AB and E any point of side AC. Let PDE be an isosceles triangle with base DE, oriented the same as ABC, and with apex angle P equal to angle A. Find the locus of all such points P.

47. PME 602 by Jack Garfunkel

Given isosceles triangle ABC and a point O in the plane of the triangle, erect directly similar isosceles triangles POA, QOB and ROC (but not necessarily similar to triangle ABC). Prove that the apexes P, Q and R of these triangle determine a triangle similar to triangle ABC.

48. PME 605 by Jack Garfunkel

Given that x is an acute angle, find the value of x if

$$\frac{\sin 4x}{2\cos 3x} = \frac{\sin x}{\cos 2x} + 2\sin x.$$

49. PME 607 by Jack Garfunkel

Triangle ABC and $A'B'C'$ are right triangles with right angles at C and C'. Prove that if $s/r > s'/r'$, then $s/R < s'/R'$, where s, s', r, r', R and R' are respectively the semiperimeters, inradii, and circumradii of ABC and $A'B'C'$.

50. PME 620 by Jack Garfunkel

A triangle ABC is inscribed in an equilateral triangle PQR. The angle bisectors of triangle ABC are drawn and extended to meet the sides of triangle PQR in points A_1, B_1, C_1. Now draw the angle bisectors of triangle $A_1B_1C_1$ to meet the sides of triangle PQR at A_2, B_2, C_2. Repeat the procedure. Prove or disprove that triangle $A_nB_nC_n$ tends to equilateral as n tends to infinity.

51. PME 629 by Jack Garfunkel
If A, B, C are the angles of a triangle, prove that

$$\cos A \cos B \cos C \leq (1 - \cos A)(1 - \cos B)(1 - \cos C).$$

52. PME 648 by Jack Garfunkel
If A, B, C are the angles of a triangle ABC, prove that

$$\prod \cos \frac{A}{2} \leq \frac{\sqrt{3}}{6} \sum \cos^2 \frac{A}{2}.$$

53. PME 656 by Jack Garfunkel
Let ABC be any triangle and extend side AB to A', side BC to B', and side CA to C' so that B lies between A and A', etc., and $BA' = \lambda \cdot AB$, $AC' = \lambda \cdot CA$, and $CB' = \lambda \cdot BC$. Find the value of λ so that the area of triangle $A'B'C'$ is four times the area of triangle ABC.

54. PME 677 by Jack Garfunkel
If A, B, and C are the angles of a triarigle, then show that

$$\frac{\cos A \cos B \cos C}{\cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}} \leq \frac{\sqrt{3}}{9}.$$

55. PME 683 by Jack Garfunkel
(a) Given three concentric circles, construct an isosceles right triangle so that its vertices lie one on each circle. (b) Is the construction always possible?

56. PME 695 by Jack Garfunkel
If ABC is a triangle, prove that

$$\sqrt{\sin A} + \sqrt{\sin B} + \sqrt{\sin C} \geq 6\sqrt{\sqrt{3} \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}.$$

57. PME 708 by Jack Garfunkel
Find a Mascheroni construction (a construction using only compasses – no straightedge allowed) for the orthic triangle of an acute triangle ABC.

58. PME 716 by Jack Garfunkel
Prove that, for $x, y, z > 0$,

$$\sqrt{xy} + \sqrt{yz} + \sqrt{zx} \geq 3\sqrt{3}\sqrt{\frac{xyz}{x+y+z}}.$$

59. SSM 4045 by Jack Garfunkel
Let a, b, and c be positive real numbers. Prove that

$$(1/a + 1/b + 1/c)(a^2 + b^2 + c^2)^{1/2} \geq 3\sqrt{3},$$

with equality if and only if $a = b = c$.

60. SSM 4069 by Jack Garfunkel
Let ABC be a triangle. Denote its altitudes by h_a, h_b, and h_c; its angle bisectors by t_a, t_b, and t_c; and the angle bisectors extended to the circumcircle of triangle ABC by T_a, T_b, and T_c. Prove that

$$T_a t_a + T_b t_b + T_c t_c = 2R(h_a + h_b + h_c),$$

where R is the circumradius of triangle ABC.

61. SSM 4072 by Jack Garfunkel
Prove that a cyclic quadrilateral with sides whose measures are consecutive positive integers cannot have an integral area.

62. SSM 4086 by Jack Garfunkel
Let triangle ABC with sides a, b, and c and inradius r be given. Consider the equilateral triangle $A'B'C'$ with sides each equal to $(a+b+c)/3$ and inradius r'. Prove that $r' \geq r$.

63. SSM 4087 by Jack Garfunkel
Consider the Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots, u_n, \ldots$, where

$$u_n = u_{n-1} + u_{n-2}, \quad \text{for } n \geq 3.$$

Prove the inequality

$$n^2 u_{n-2} \geq \sum_{k=1}^{n} k u_k, \quad \text{for } n \geq 5.$$

64. SSM 4115 by Jack Garfunkel
Squares are constructed (outwardly) on each side of a parallelogram. Prove that the centers of these squares are the vertices of a square.

65. SSM 4122 by Richard A. Gibbs, L\'aszl\'o Sz\H{u}cs, and Jack Garfunkel

Given teo circles K_1 and K_2 with radii r_1 and r_2 respectively, each circle being exterior to the other:

(a) Characterize the set S consisting of all points P which lie exterior to both K_1 and K_2 and which have the following property:

There exists an isosceles right triangle PQR with right angle at P such that Q is on K_1 and R is on K_2.

(b) Given $P \in S$, describe a construction of the triangle PQR satisfying the conditions stated in (a).

66. SSM 4140 by Jack Garfunkel

In triangle ABC, m_a, m_b, and m_c denote the medians to sides a, b, and c and M_a, M_b, and M_c denote the medians extended to the circumcircle of ABC. Prove that

$$M_a m_a + M_b m_b + M_c m_c = a^2 + b^2 + c^2.$$

67. SSM 4150 by Jack Garfunkel

Solve for $x \in [0, 2\pi)$:

$$(16/81)^{\sin^2 x} + (16/81)^{\cos^2 x} = 26/27.$$

68. SSM 4152 by Jack Garfunkel

In a triangle ABC, the cevians AD, BE, and CF concur at a point P, and $\angle FDB = \angle EDC$. Prove that AD is the altitude to the side BC.

69. SSM 4165 by Jack Garfunkel

Let $ABCDEFG$ be a regular heptagon. Prove that

$$\frac{1}{CD} = \frac{1}{AC} + \frac{1}{AD}.$$

70. SSM 4173 by Jack Garfunkel

Let D, E, and F be the points of tangency of the incircle of triangle ABC to the sides BC, CA, and AB, respectively. Prove that

$$\text{perimeter of } \triangle DEF \leq \frac{1}{2} \text{ perimeter of } \triangle ABC$$

71. SSM 4187 by Jack Garfunkel

Given two externally tangent circles with radii $r_1 < r_2$. If θ is the angle between the common external tangents, show that

$$\sin\theta = \frac{4(r_2 - r_1)\sqrt{r_1 r_2}}{(r_1 + r_2)^2}.$$

72. SSM 4192 by Jack Garfunkel
If A, B, C are the angles of a triangle, prove that

$$\sin^3 A + \sin^3 B + \sin^3 C \geq 3 \sin A \sin B \sin C.$$

73. SSM 4211 by Jack Garfunkel
Erect equilateral triangles $AC'B$, $BA'C$, $CB'A$ externally on the sides AB, BC, CA, respectively, of an arbitrary triangle ABC. The lines AA', BB', CC' meet at F, known as the Fermat point of $\triangle ABC$. Prove that $\triangle A'B'C'$ has the same Fermat point as $\triangle ABC$.

74. SSM 4222 by Jack Garfunkel
Let D, E, F be the points of tangency of the excircles of triangle ABC to the sides BC, CA, AB, respectively. (The lines AD, BE, CF are concurrent at a point N, called the Nagel point.) Prove that if $BE = CF$ then the triangle ABC is isosceles.

75. SSM 4231 by Jack Garfunkel
Let E be the point of intersection of the diagonals of a quadrilateral $ABCD$. Let P, Q, R, S be the feet of the perpendiculars from E to the sides AB, BC, CD, DA, respectively. Prove that the perimeter of $PQRS$ is less than or equal to $AC + BD$.

76. SSM 4240 by Jack Garfunkel
Let α, β, γ be the angles of a triangle ABC. Prove that

$$9 \sin(\alpha/2) \sin(\beta/2) \sin(\gamma/2) - \cos\alpha \cos\beta \cos\gamma \leq 1,$$

with equality if and only if the triangle ABC is equilateral.

77. SSM 4249 by Jack Garfunkel
Given a quadrilateral $ABCD$, extend each side by its own length to points A', B', C', D'. The points B, C, D, A are midpoints of the line segments AB', BC', CD', DA', respectively. Prove that the area of the quadrilateral $A'B'C'D'$ is five times the area of $ABCD$.

78. SSM 4256 by Jack Garfunkel
Prove that, if A, B, C are the angles of an acute triangle, then

$$1/\sqrt{\cos A} + 1/\sqrt{\cos B} + 1/\sqrt{\cos C} \geq 3\sqrt{2}$$

with equality if and only if the triangle is equilateral.

79. SSM 4261 by Jack Garfunkel

Let h_a, h_b, h_c be the altitudes of an acute-angled triangle ABC, and let H_a, H_b, H_c denote the altitudes extended to the circumcircle. Prove that $H_a/h_a + H_b/h_b + H_c/h_c = 4$.